TASHKENT UNIVERSITY OF
INFORMATION TECHNOLOGIES
NAMED AFTER MUHAMMAD AL-KHWARIZMI

MUHAMMAD AL-XORAZMIY NOMIDAGI
TOSHKENT AXBOROT TEXNOLOGIYALARI
UNIVERSITETI

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Zarnigor Fayzullayeva, Sadullayeva Shakhlo Azimboyevna № 3(13)2023

Numerical modeling of a doubly nonlinear parabolic reaction-diffusion process

Zarnigor Fayzullayeva
Tashkent University of Information Technologies named after Muhammad al-Khwarizmi
Tashkent, Uzbekistan
akmalaliqulov@gmail.com

Sadullayeva Shakhlo Azimboyevna
Joint Belarusian-Uzbek Intersectoral Institute of Applied Technical Qualifications in Tashkent
orif_sh@list.ru

Abstract— In this article, the limitation of the rate of heat propagation, spatial localization properties of mathematical models of heat propagation and diffusion processes described by the system of nonlinear equations of parabolic type in two-component environment are defined and properties of nonlinear mathematical model of convective migration rate depending on time and damping coefficient in two-component medium were studied. The effect of these factors on the studied heat transfer and diffusion processes is shown. Analyzing the solution based on the principle of comparison, it was determined that the one-way localization of the solution and the effect of convective transfer occur under the influence of convective transfer in two dimensions. The main calculation scheme of diffusion processes in moving media characterized by the system of parabolic-type nonlinear equations was built, the algorithm and methods and software were developed and the solution was visualized

***Keywords— boundary, technology, polytrophic filtration, auto model, model mathematical, diffusion processes, reaction-diffusion, numerical solutions*.**

INTRODUCTION

The study of nonlinear mathematical models is one of the urgent problems of developing methods of increasing the efficiency of the reaction-diffusion process control system in world practice in the field of natural sciences. According to the database of the Elsevier Analytical Center, the number of scientific works of world scientists devoted to nonlinear filtering and biological population in solving the Cauchy problem for the nonlinear reaction-diffusion equation in the boundary condition and applying its results to practice is steadily increasing. In this field, scientists from the developed countries of the world, including the USA, the Russian Federation, Japan, Azerbaijan, Turkey, China and other countries, have studied and researched the results of nonlinear reaction-diffusion equations and systems of equations, as well as the Cauchy problem for parabolic-type problems and the boundary value for these equations. focused on the practical application of the issues.

In recent years, a number of fundamental problems in the world are related to mathematical modeling of nonlinear processes, improvement of visualization methods and tools, implementation of important results of solutions of double nonlinear reaction-diffusion problems. Currently, conducting scientific research in the following directions in connection with the study and application of double nonlinear equations is considered one of the important tasks: development of visualization methods as a result of the study of nonlinear models; create complex programs that help to study non-linear processes; special attention is being paid to the creation of the technology of conducting a computational experiment, the creation of a method of controlling the evolution process over time, the development of a computerized system for determining the properties of parameters based on their dynamic change.

The article considers the Cauchy problem for the following system of nonlinear parabolic equations.

$$\begin{cases} \frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(\left|\frac{\partial u^{k_1}}{\partial x}\right|^{p-2}\frac{du^{m_1}}{dx}\right) \pm cv_1(t)\frac{\partial y}{\partial x} - b_1 u^{q_1}\left|\frac{\partial v^{m_1}}{\partial x}\right|^{p_1} \\ \frac{\partial v}{\partial t} = \frac{\partial}{\partial x}\left(\left|\frac{\partial v^{k_2}}{\partial x}\right|^{p-2}\frac{dv^{m_2}}{dx}\right) \pm cv_1(t)\frac{\partial v}{\partial x} - b_1 v^{q_2}\left|\frac{\partial u^{m_2}}{\partial x}\right|^{p_2} \end{cases} \quad (1)$$

$$u(x,0) = u_0(x) \geq 0 \text{ ,} v(x,0) = v_0(x) \geq 0 \text{ , } x \in R \quad (2)$$

Where, $m_i \geq 1$, $p \geq 2$, $q_i > 0$, $k_i \geq 1$, (i=1,2), numerical parameters of the environment, $u_0(x)$ *va* $v_0(x)$- positive definite continuous carrier functions (ambient temperature).

All processes occurring in nature are represented by non-linear problems. The considered equation, in particular, the value of the numerical parameters, consists of well-known equations, including: porous environment equations, when $p = 2$ Buseneski equations, $m = 2, p = 2$ Leibenson equations are used to describe the movement of groundwater, $k = m$ is called P-Laplace equations.

(1) system of equations, we observe possible phenomena in various fields such as physics, chemistry, biology, ecology. There are many problems that manifest themselves in reaction-diffusion systems [5-7].

The boundary problem (1), (2) plays an important role in mathematical modeling of the diffusion process in a non-linear medium, fluid flow in porous environment, biological population dynamics, polytropic filtration, synergetics and solving problems in a number of other fields. For example, $u(x,t)$ and $v(x,t)$ represent the density of two biological populations or the temperature of two-component porous environment in the process of heat propagation [1-4].

(1), (2) boundary value problem plays an important role in mathematical modeling of the diffusion process in non-linear media, fluid flow in porous media, biological population dynamics, polytropic filtration, synergetics, and solving problems in a number of other areas. For instance, $u(x,t)$ and $v(x,t)$ represent the two biological population density or the temperature of the two porous environment in the process of heat dissipation [1-4].

As is known, the system of equations (1) is degenerate in the domain $u, v \equiv 0$ and does not have a classical solution. In this case, the generalized solution of problem (1)-(2) is studied as follows:

The given system of equations (1) are degenerate in k(p-2)+m-1>0, so its solution in the Q ={(t, x): 0 < t <T, } domain (1) is understood as a satisfactory generalized solution in distribution of equations. The system (1) represents a number of processes, and is considered as a mathematical model of reaction-diffusion, heat diffusion, filtration, and etc. The given equations (1) describe the slow diffusion process at k(p-2)+m-1>0, and the fast diffusion process at k(p-2)+m-1<0.

It is known that the solution of problem (1), (2) is global or unbounded under specific conditions of numerical parameters. Regarding (1) and (2), several scientists have conducted research with this very question. Including:

Z. Rakhmonov [5] observed the issue of heat conduction in an inhomogeneous environment:

$$\rho_1(x)\frac{\partial u}{\partial t} = div\left(\left|\nabla u^l\right|^{p-2}\nabla u^l\right) + \rho_2(x)u^q \quad x \in R^N \quad t > 0 \quad (3)$$

$$u(x,0) = u_0(x) \qquad x \in R^N \quad (4)$$

Here:

$$(\cdot) = grad_x(\cdot) \equiv \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \dots, \frac{\partial}{\partial x_N}\right)(\cdot), \rho_1(x) = |x|^m,$$
$$\rho_2(x) = |x|^n$$

It is considered as a model of the combustion process in the medium where in n=m=0 power of heat release is $u^q \geq 0$ and temperature depends on $u = u(x,0) \geq 0$. Auto model solutions of problems (3), (4) were constructed by the author and asymptoreattics of auto model solutions were found.

Sh. Sadullayeva [6,7] studied the issue of heat propagation in the following non-homogeneous medium in the $Q_T = \{(t,x): 0\} < t < T, x \in R^N\}$ domain:

$$Au \equiv -\frac{\partial(\rho(x)u)}{\partial t} + \nabla(u^{m-1}\left|\nabla u^k\right|^{p-2}\nabla u^l) \quad (5)$$

$$u\big|_{t=0} = u_0(x) \geq 0, x \in R^N \quad (6)$$

Equations (11), (12) are considered as models of liquid and gas filtration, heat diffusion processes with a double nonlinear coefficient. Diffusion in the medium, where $\rho(x)$ is the density of the medium. In this paper, numerical solutions of problems (5), (6) were built based on the auto model approach and localization conditions were found.

Introduce the following notations to find the automodel solution for the system of equations (1), (2).

$$\begin{cases} u(t,x) = \omega(t,\eta(x,t)) \\ v(t,x) = z(t,\eta(x,t)) \end{cases}$$

$$\begin{cases} u_t = w_t + w_\eta \cdot \eta_t \\ v_t = z_t + z_\eta \cdot \eta_t \end{cases} \quad (7)$$

$$\eta(x,t) = x \pm c\int v_1(t)\,dt = x \pm c\int_{t_0}^{t} v_1(t)\,dt + c_1$$

$$\eta_x = 1$$

$$\begin{cases} w(t,\eta) = f(\xi) \\ z(t,\eta) = g(\xi) \end{cases} \quad (8)$$

$$\xi = \eta \cdot t^{-\frac{1}{p}} = t^{-\frac{1}{p}} \cdot \left[x \pm c\int_{t_0}^{t} v_1(t)dt \right]$$

$$\begin{cases} -\frac{\xi}{pt} f_\xi = \frac{1}{t}\left(\left| f_\xi^{k_1}\right|^{p-2} f_\xi^{m_1}\right)_\xi - t^{-\frac{p_1}{p}} b_1 f^{q_1}\left|g_\xi^m\right|^{p_1} \\ -\frac{\xi}{pt} g_\xi = \frac{1}{t}\left(\left|g_\xi^{k_2}\right|^{p-2} g_\xi^{m_2}\right)_\xi - t^{-\frac{p_2}{p}} b_2 g^{q_2}\left|g_\xi^m\right|^{p_2} \end{cases}$$

$$w_t = f(\xi) \qquad \xi = \eta \cdot t^{-\lambda} \qquad z = g(\xi) \qquad \xi = \eta \cdot t^{-\lambda}$$

$$\lambda = \frac{1}{p}$$

As a result, the system of equations (1) comes to the automodel equation of the following form:

$$\begin{cases} \frac{d}{d\xi}\left(\left|\frac{df^{k_1}}{d\xi}\right|^{p-2}\frac{df^{m_1}}{d\xi}\right) + \frac{1}{p}\xi\frac{df}{d\xi} - b_1 f^{q_1}\left|\frac{dg^{m_1}}{d\xi}\right|^{p_1} = 0 \\ \frac{d}{d\xi}\left(\left|\frac{dg^{k_2}}{d\xi}\right|^{p-2}\frac{dg^{m_2}}{d\xi}\right) + \frac{1}{p}\xi\frac{dg}{d\xi} - b_2 g^{q_2}\left|\frac{df^{m_2}}{d\xi}\right|^{p_2} = 0 \end{cases} \quad (9)$$

For the asymptotics of the solution of equation (1), let's consider the following function

$$\begin{cases} \overline{f}(\xi) = A_1\left(a_1 - \xi^{\gamma_0}\right)^{\gamma_1} \\ \overline{g}(\xi) = A_2\left(a_2 - \xi^{\gamma_0}\right)^{\gamma_2} \end{cases}$$

$$\begin{cases} \gamma_0 = \frac{1-p}{p-2} \\ \gamma_1 = \frac{p-1}{m_1(p-1)-1} \\ \gamma_2 = \frac{p-1}{m_1(p-1)-1} \end{cases} \cdot a_i = const \geq 0$$

$$i = \overline{1,2..n}$$

$$A_1 = \left[\frac{(\gamma_0\gamma_1)^{1-p}\, k_1^{2-p}}{pm_1}\right]^{\frac{\gamma_1}{p-1}}$$

$$A_2 = \left[\frac{(\gamma_0\gamma_2)^{1-p}\, m_2^{2-p}}{pm_2}\right]^{\frac{\gamma_2}{p-1}}$$

Numerical solution. For this, build a net according to the spatial coordinate

$$S_h = \{x_i = i \cdot h, h > 0, i = 1, 2...., n, n \cdot h = b\}$$

and according to time

$$V_h = \{x_i = i \cdot h, h > 0, i = 1, 2...., n, n \cdot h = b\}$$

Write the equation (1) in the following form:

$$\begin{cases} \dfrac{\partial u}{\partial t} = \dfrac{\partial}{\partial x}\left(K(u)\dfrac{\partial u}{\partial x}\right) \pm cv_1(t)\dfrac{\partial u}{\partial x} - b_1 u^{q_1}\left|\dfrac{\partial v^{m_1}}{\partial x}\right|^{p_1}, \\ \dfrac{\partial v}{\partial t} = \dfrac{\partial}{\partial x}\left(B(\vartheta)\dfrac{\partial u}{\partial x})\right) \pm cv_1(t)\dfrac{\partial v}{\partial x} - b_1 v^{q_2}\left|\dfrac{\partial u^{m_2}}{\partial x}\right|^{p_2}, \end{cases}$$

$$K(u) = u^{(m_1-1)(p_1-1)}\left|\frac{\partial u}{\partial x}\right|^{p_1-1}$$

$$B(\vartheta) = \vartheta^{(m_2-1)(p_2-1)}\left|\frac{\partial \vartheta}{\partial x}\right|^{p_2-1}$$

we use a simple iteration method to solve the system of equations and get:

$$\begin{cases} a_i(y) = B_1\left(\dfrac{y_i^j - y_{i-1}^j}{2}\right) \\ a_i(g) = B_2\left(\dfrac{g_i^j - g_{i-1}^j}{2}\right) \end{cases}$$

Diverse iteration for solving a system of nonlinear equations we use methods and obtain the following

$$\begin{cases} \dfrac{u_i^{j+1,s+1} - u_i^{j,s}}{\tau} = \dfrac{1}{h^2}\left(a_{i+1}^{s}(u)\left(u_{i+1}^{j+1,s+1} - u_i^{j+1,s+1}\right) - a_i^{s}(u)\left(u_i^{j+1,s+1} - u_{i-1}^{j+1,s+1}\right)\right) + c_1 v_1(t)\dfrac{u_i^{j+1,s+1} - u_{i-1}^{j+1,s+1}}{h} - G_1\left(u_i^{s}, v_i^{s}\right) \\ \dfrac{v_i^{j+1,s+1} - v_i^{j,s}}{\tau} = \dfrac{1}{h^2}\left(a_{i+1}^{s}(v)\left(v_{i+1}^{j+1,s+1} - v_i^{j+1,s+1}\right) - a_i^{s}(v)\left(v_i^{j+1,s+1} - v_{i-1}^{j+1,s+1}\right)\right) + c_2 v_2(t)\dfrac{v_i^{j+1,s+1} - v_{i-1}^{j+1,s+1}}{h} - G_2 \end{cases}$$

Accuracy of repetition in calculation according to the repetition schemeis given and the process continues until the following condition is met

$$\max_{0\le i\le n}\left| u_i^{s+1} - u_i^{s}\right| < \varepsilon,\ \max_{0\le i\le n}\left| v_i^{s+1} - v_i^{s}\right| < \varepsilon$$

To numerically solve a system of algebraic equationsthe spark plug method is used. according to the pragonka method

$$\begin{cases} \bar{u}_i = \alpha_{1i}\left(\beta_{1i} + \bar{u}_{i+1}\right), \\ \bar{v}_i = \alpha_{2i}\left(\beta_{2i} + \bar{v}_{i+1}\right). \end{cases}$$

here are the asymptotics where the initial approximation is found

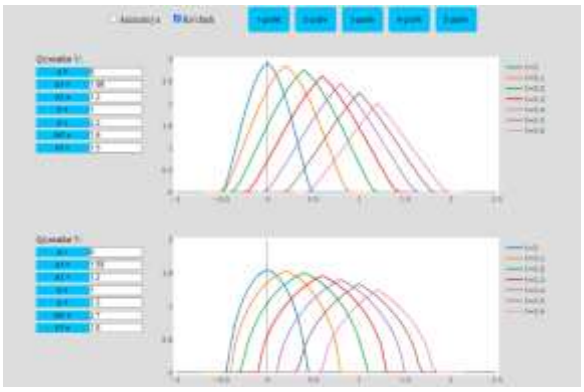


Figure 1. Numerical solution of problem 1
a1 = 1.85, A1 = 1.2, b1 = 1.5, p = 2.1, m1 = 1.6, k1 = 1.5

a2 = 1.85, A2 = 1.2, b2 = 1.5, p = 2.1, m2 = 2.7, k2 = 1.8

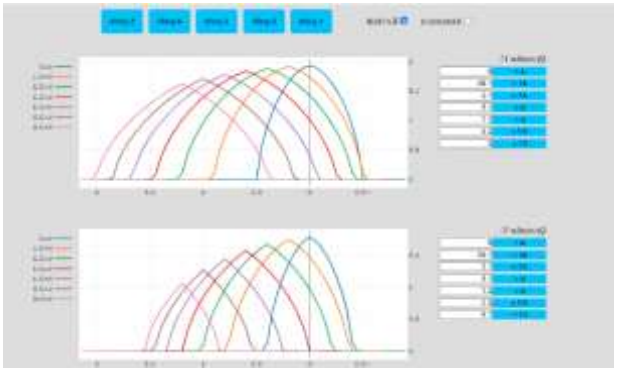


Figure 2. Numerical solution of problem 1
a1 = 1.85, A1 = 1.2, b1 = 1.5, p = 2.1, m1 = 2.4, k1 = 2

a2 = 1.85, A2 = 1.2, b2 = 1.5, p = 2.1, m2 = 2.3, k2 = 1.9

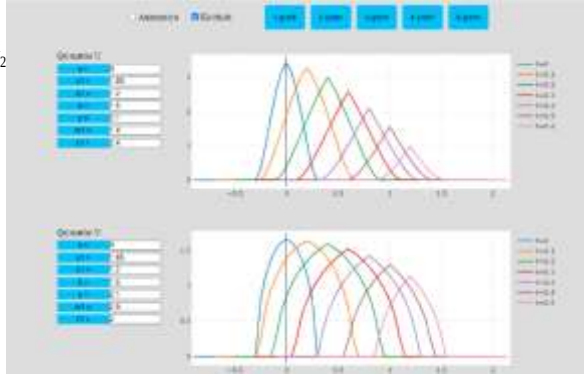


Figure 2. . Numerical solution of problem 1
a1 = 1.85, A1 = 1.2, b = 1.5, p = 2.1, m1 = 1.4,

k1 = 2.4

a1 = 1.85, A1 = 1.2, b = 1.5, p = 2.1, m1 = 2.9,

k1 = 2



Figure 4. . Numerical solution of problem 1
a1 = 1.85, A1 = 1.2, b1 = 1.5, p = 2.1, m1 = 1.6, k1 = 1.5

a2 = 1.85, A2 = 1.2, b2 = 1.5, p = 2.1, m2 = 2.7, k2 = 1.8

CONCLUSION

The obtained results made it possible to solve the problem of initial approximation of the numerical solution of the problem through the iteration method. The emergence of important situations that change the behavior of the result is indicated. The properties of the nonlinear mathematical model on the effect of convective displacement were studied, its speed depends on time and the damping coefficient also depends on time. The effects of these factors on the studied heat transfer, diffusion, filtration phenomena in gases and liquids, and the spread of viruses are shown.

Analyzing the solution according to the principle of comparison, it was found that the phenomenon of one-dimensional localization of the solution under the influence of convective transfer and the effect of convective transfer appear under the influence of convective transfer, its speed. time function, and spatial localization and solution can lead to asymmetry.

Based on the auto model analysis, the condition and boundary value of the Fujita-type global solution of the Cauchy problem for the system of parabolic-type double-linear nonlinear equations under the effect of damping were obtained. This result covers, in particular, the results of H. Fujit, A. A. Samarsky, S. P. Kurdyumov, V. A. Galaktionov, A. P. Mikhailov, and also M. Aripov.

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